Published by Institute of Physics Publishing for SISSA/ISAS
Received: *August 1, 2002*
Accepted: *September 2, 2002*

Gravity-trapping domain walls from resolved branes*

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Abstract: All previous Randall-Sundrum type models have required a Z_2 identification source which does not have a known string theoretic origin. We show that the near-horizon of various resolved branes on an Eguchi-Hanson instanton dimensionally reduce to a five-dimensional domain wall that traps gravity, without an additional delta-function source. This brings us substantially closer to embedding infinite extra dimensions in M-theory. Also, this provides us with a brane world model for a strongly-coupled Yang-Mills field theory with quark-antiquark charge screening at finite separation distance.

Keywords: Extra Large Dimensions, p-branes, Classical Theories of Gravity.

*This work is supported in part by the Francqui Foundation (Belgium), the Actions de Recherche Concertées of the Direction de la Recherche Scientifique - Communauté Francaise de Belgique, IISN-Belgium (convention 4.4505.86).

JHEP09(2002)001

Contents

1. Introduction

Randall and Sundrum [1, 2] have shown that, with fine tuned brane tension, a flat 3-brane embedded in AdS_5 can have a single massless bound state. Four-dimensional gravity is recovered at low-energy scales. Five-dimensional domain walls which localize gravity may arise from a sphere reduction from ten or eleven dimensions, as the near-horizon of extremal p-branes [3]. However, in order to localize a massless graviton state, one must add a delta-function source which comes about from imposing Z_2 symmetry. As of yet, this source has no known origin in the ten or eleven-dimensional theory. This has prevented gravity-trapping domain walls from fully arising in the context of string theory.

From the type IIB perspective, in which the brane world scenario could arise from the near-horizon of a D3-brane, it has been proposed that the Z_2 identification source could arise from D7-branes.[1] If the spherical symmetry of the transverse space is relaxed, one could wrap D7-branes around a 4-cycle of the five-dimensional compact space [4]. However, the D7-charge would need to be canceled. One candidate for this is an orbifold charge associated with the base of an elliptically fibered F-theory Calabi-Yau (complex) 4-fold [6]. In this case, the dimensions transverse to the D3-brane are necessarily compact.

In the present paper, we will also relax the spherical symmetry of the transverse space. Our goal is to find a p-brane interpretation of a Randall-Sundrum brane world in an infinite extra dimension, without having to add a Z_2 identification source. In particular, we will consider the replacement of the standard flat transverse space by a smooth space of special holonomy,[2] preserving a fraction of the original supersymmetry. In general, however, the resulting solution is singular.

[1] It has also been proposed that the AdS_4 brane in AdS_5 can be realized in ten dimensions as D5-branes in the near-horizon of D3-branes [5].

[2] A Ricci-flat space with fewer covariantly constant spinors.

In certain cases, deformations of the standard brane solutions can have the effect of "resolving" such singularities. These deformations are the result of an additional flux on a Ricci-flat space transverse to the brane. Since this resolution can break additional supersymmetry, such non-singular solutions may serve as viable gravity duals of strongly-coupled Yang-Mills field theories with less than maximal supersymmetry [7]–[13].

In the IR (small r) regime, the bound-state spectrum of a minimally-coupled scalar (dilaton) corresponds to the spectrum of the operator $\mathrm{Tr}\, F^2$ on the gauge theory side, thus providing information about the glueball spectrum. The spectrum of a minimally-coupled scalar in the background of a resolved heterotic 5-brane on an Eguchi-Hanson instanton is continuous above a mass gap. Boundary conditions eliminate bound states of non-positive mass [13]. This type of spectrum is associated with a complete screening of charges in a quark-antiquark pair at a finite separation distance.

A minimally-coupled scalar may also be associated with the graviton fluctuations polarized along the brane worldvolume. In this case, with appropriate boundary conditions, there is a massless bound state as well as a continuous spectrum above a mass gap. This provides a ten-dimensional supergravity solution whose dimensional reduction is a gravity-trapping domain wall. It should be emphasized that no additional delta-function source from Z_2 identification is needed to localize the massless graviton state. The above also applies for a D4-brane on $S^1 \times M_4$ and a D3-brane on $T^2 \times M_4$, where M_4 is the Eguchi-Hanson instanton.

This paper is organized as follows. In section 2, we show how a resolved heterotic 5-brane, D4-brane and D3-brane on an Eguchi-Hanson instanton give rise to the same five-dimensional domain wall. In section 3, we show that this domain wall has a bound massless graviton state with a continuous Kaluza-Klein spectrum above a mass gap. We derive the modified newtonian potential. In section 4, we provide concluding remarks.

2. Five-dimensional domain wall from resolved branes on Eguchi-Hanson instanton

We will briefly describe the various resolved branes on an Eguchi-Hanson instanton. Derivations and details for the case of the heterotic 5-brane and D4-brane are provided in [12] and for the D3-brane in [14, 15].

2.1 Heterotic 5-brane

A deformed heterotic 5-brane on an Eguchi-Hanson instanton is given by

$$\begin{aligned} ds_{10}^2 &= H^{-1/4}(-dt^2 + dx_j^2) + H^{3/4} ds_4^2\,, \\ \mathrm{e}^{-\phi} * F_3 &= d^6x \wedge dH^{-1}\,, \qquad \phi = \frac{1}{2}\log H\,, \qquad F_{(2)} = mL_{(2)}\,, \end{aligned} \tag{2.1}$$

where $j = 1, \ldots, 5$. The metric for the Eguchi-Hanson instanton is

$$ds_4^2 = W^{-1}dr^2 + \frac{1}{4}r^2 W(d\psi + \cos\theta d\phi)^2 + \frac{1}{4}r^2 d\Omega_2^2\,, \tag{2.2}$$

where

$$W = 1 - \frac{a^4}{r^4}\,, \tag{2.3}$$

and $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$. The radial coordinate has the range $a \le r \le \infty$. $F_{(2)}$ is provided by an abelian U(1) field in the transverse space, and acts as a Chern-Simons type term:

$$dF_{(3)} = \frac{1}{2} F_{(2)} \wedge F_{(2)}\,. \tag{2.4}$$

In order to satisfy the equations of motion,

$$\Box H = -\frac{1}{4} m^2 L_{(2)}^2\,. \tag{2.5}$$

We consider H depending only on r. $L_{(2)}$ must be a self-dual harmonic 2-form, which has been found to be

$$L_{(2)} = r^{-3} dr \wedge (d\psi + \cos\theta d\phi) + \frac{1}{2} r^{-2} \sin\theta d\theta \wedge d\phi\,. \tag{2.6}$$

Since this harmonic function is normalizable, the previously-mentioned singular term in H can be canceled. (2.5) becomes

$$\frac{1}{r^3}\partial_r r^3 W \partial_r H = -\frac{4m^2}{r^8}\,, \tag{2.7}$$

yielding

$$H = 1 + \frac{m^2 + a^4 b}{4a^6} \log\left(\frac{r^2 - a^2}{r^2 + a^2}\right) + \frac{m^2}{2a^4 r^2}\,. \tag{2.8}$$

Choosing the integration constant $b = -m^2/a^4$, the logarithmic term corresponding to a naked singularity at $r = a$ cancels, so that

$$H = 1 + \frac{R^2}{r^2}\,, \tag{2.9}$$

where $R^2 = m^2/(2a^4)$. Thus, the singularity has been resolved. This solution provides a gravity dual of $N = 2$ six-dimensional field theory. In the IR (small r) regime, the bound-state spectrum of a minimally-coupled scalar (dilaton) corresponds to the spectrum of the operator $\mathrm{Tr}\, F^2$ on the gauge theory side, thus providing information about the glueball spectrum. The spectrum of a minimally-coupled scalar in the background of a resolved heterotic 5-brane on an Eguchi-Hanson instanton is continuous above a mass gap. Boundary conditions eliminate non-positive bound states. Such a spectrum is associated with a complete screening of charges in a quark-antiquark pair at a finite separation distance which is inversely proportional to the mass gap [17].

We will consider the five-dimensional domain wall resulting from a reduction over the angular coordinates of the transverse space and two coordinates on the world volume. We can use the following Ansatz for the reduction on S^n [3]:

$$ds_D^2 = \mathrm{e}^{-2\alpha\phi} ds_d^2 + g^{-2} \mathrm{e}^{\frac{2(d-2)}{n}\alpha\phi} d\Omega_n^2\,, \tag{2.10}$$

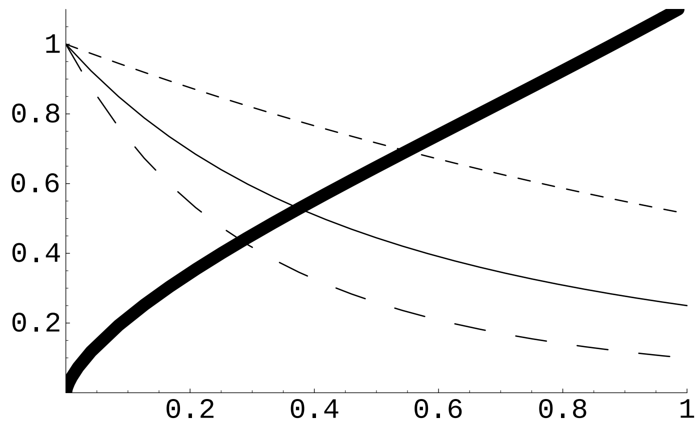

Figure 1: Conformal factors for various five-dimensional domain walls.

where

$$\alpha = -\sqrt{\frac{n}{2(d-2)(D-2)}}\,. \tag{2.11}$$

A reduction on the ψ, θ and ϕ coordinates yields a BPS domain wall in $D = 7$ gauged supergravity, which preserves 1/4 of the original supersymmetry. Toroidally reducing over x_4 and x_5 does not change the supersymmetry of the solution. Thus, reducing the resolved heterotic 5-brane over $T^3 \times S^2$ (5-brane wrapped on T^2) yields, in the decoupling limit ($H \to R^2/r^2$), the following five-dimensional domain wall solution:

$$ds_5^2 = \left(\frac{r}{R}\right)^{4/3} W^{1/3}(-dt^2 + dx_i^2) + \left(\frac{R}{r}\right)^{2/3} W^{-2/3} dr^2\,, \tag{2.12}$$

where $i = 1, 2, 3$. If we make the coordinate transformation

$$\frac{r}{a} = \cosh^{1/2}(kz)\,, \tag{2.13}$$

then the metric (2.12) can be written in the conformally-flat frame as

$$ds_5^2 = \sinh^{2/3}(kz)(-dt^2 + dx_i^2 + dz^2)\,. \tag{2.14}$$

$z = 0$ corresponds to $r = a$. This differs from most brane world scenarios arising from p-brane origins, in that the z and r coordinates are usually inversely related. A result of this difference is shown in figure 1. The conformal factors arising from the near-horizon of a D3-brane (normal line), D4 or M5-brane (large dashing) and D5-brane or M5/M5-brane intersection [3] (small dashing) are a positive finite value at $z = 0$ and decrease asymptotically to zero as z gets larger. On the other hand, the conformal factor for the near-horizon of the resolved heterotic 5-brane on an Eguchi-Hanson instanton (bold line) vanishes at $z = 0$ and increases as z gets larger. As we will show, the result of this behavior is an effective potential that has an infinite well at $z = 0$ and is capable of having a bound massless graviton state.

2.2 D4-brane

The metric of the D4-brane solution of type IIA supergravity is given by

$$ds_{10}^2 = H^{-3/8}(-dt^2 + dx_k^2) + H^{5/8} ds_5^2\,, \tag{2.15}$$

where $k = 1, \ldots, 4$. We take the transverse space to be $M_5 = M_4 \times S^1$, and we can take M_4 to be the Eguchi-Hanson instanton. This transverse space can support a normalizable self-dual harmonic 2-form, as in the case of the heterotic 5-brane. Thus, as before, the singularity can be resolved and the corresponding H is given by (2.9). Reducing the resolved D4-brane on the angular coordinates in M_5 as well as the worldvolume coordinate x_4 ($T^3 \times S^2$, with the D4-brane wrapped on S^1), we obtain in the decoupling limit a regular domain wall in $D = 5$, with the same metric as in the case of the heterotic 5-brane, given by (2.12). Again, we express the metric in the conformally-flat frame given by (2.14).

2.3 D3-brane

The metric of the D3-brane solution of type IIB supergravity is given by

$$ds_{10}^2 = H^{-1/2}(-dt^2 + dx_i^2) + H^{1/2} ds_6^2\,, \tag{2.16}$$

where $i = 1, 2, 3$. We take the transverse space to be [14, 15]

$$ds_6^2 = dz d\bar{z} + ds_4^2\,, \tag{2.17}$$

where ds_4^2 is the metric of the Eguchi-Hanson instanton given by (2.2) and the complex coordinate $z = x_4 + ix_5$. The self-dual five-form field strength supporting the D3-brane must obey the equation of motion

$$d * F_{(5)}^{RR} = -F_{(3)}^{NS} \wedge F_{(3)}^{RR}\,, \tag{2.18}$$

where a deforming three-form flux provides the Chern-Simons type term. This three-form can be expressed in terms of a harmonic function γ in $R^2 \sim C$ and an anti-self-dual harmonic two form in the Eguchi-Hanson instanton space. The details of this are provided in [14, 15]. As long as there is no source term from a boundary action, we can consider H to depend only on r. In this case, H has the form (2.8) with $m^2 \to a^4/(2\pi^2)$. As in the previous cases, the singularity can be resolved and the corresponding H is given by (2.9).

Reducing the resolved D3-brane on the angular coordinates in the Eguchi-Hanson instanton space as well as x_4 and x_5 ($T^3 \times S^2$), we obtain in the decoupling limit a regular domain wall in $D = 5$, with the same metric as in the two previous cases, given by (2.12). As before, we express this metric in the conformally-flat frame given by (2.14).

3. Localization of graviton

The equation of motion for a graviton fluctuation is

$$\partial_M \sqrt{-g} g^{MN} \partial_N \Phi = 0\,. \tag{3.1}$$

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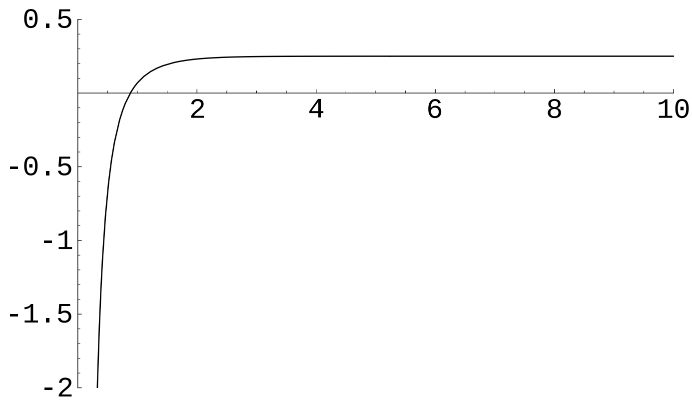

Figure 2: $V(z)$ versus kz.

We take $\Phi = \phi(z)M(t,x_i)$, where $\Box_{(4)}M = m^2M$ and $\Box_{(4)}$ is the laplacian on t, x_i. For the background (2.14) the radial wave equation is

$$-\frac{1}{\sinh(kz)}\partial_z \sinh(kz)\partial_z\phi = m^2\phi\,. \tag{3.2}$$

With the wave function transformation

$$\phi = \sinh^{-1/2}(kz)\psi\,, \tag{3.3}$$

the wave equation (3.2) becomes

$$-\partial_z^2\psi + V(z)\psi = m^2\psi\,, \tag{3.4}$$

where the effective potential

$$V(z) = \frac{k^2}{4}(1-\sinh^{-2}(kz))\,. \tag{3.5}$$

As shown in figure 2, even without introducing an additional delta-function source, $V(z)$ is similar to the effective potential for the case of a gravity-trapping domain wall derived from a D5-brane on a standard flat transverse space [3], which is

$$\tilde{V}(r) = \frac{k^2}{4} - k\delta(z)\,. \tag{3.6}$$

The graviton spectrum has a mass gap of

$$m_{\rm gap}^2 = V(z\to\infty) = \frac{k^2}{4}\,. \tag{3.7}$$

This separation between the massless graviton and the massive modes better ensures that we have a well-defined effective field theory [16]. Other gravity-trapping domain walls whose spectrum has a mass gap include that associated with a D5-brane reduced on $T^2\times S^3$ [3] and a D3-brane distributed over a disk [16]. In all cases, the mass gap results, in the p-brane perspective, from the harmonic function H supported by a four-dimensional transverse space.

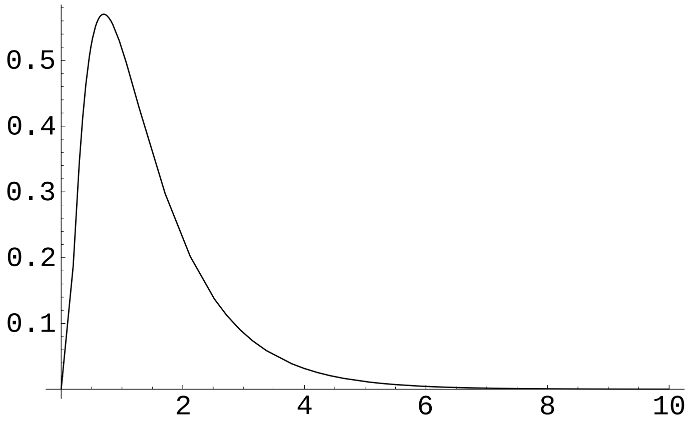

Figure 3: $\psi_0(z)$ versus kz.

The wave function solution is

$$\psi_m = N_m \sinh^{-i\gamma}(kz) F\left[\frac{1}{4} - i\frac{\sqrt{3}}{4} + i\frac{\gamma}{2}, \frac{1}{4} + i\frac{\sqrt{3}}{4} + i\frac{\gamma}{2}, 1 + i\gamma, -\sinh^{-2}(kz)\right] + \\ + M_m \sinh^{i\gamma}(kz) F\left[\frac{1}{4} - i\frac{\sqrt{3}}{4} - i\frac{\gamma}{2}, \frac{1}{4} + i\frac{\sqrt{3}}{4} - i\frac{\gamma}{2}, 1 - i\gamma, -\sinh^{-2}(kz)\right], \quad (3.8)$$

where $F[a, b, c, y]$ is the solution to the hypergeometric equation

$$y(1-y)F''(y) + [c - (a+b+1)y]F'(y) - abF(y) = 0\,, \quad (3.9)$$

and $\gamma = \sqrt{(m/k)^2 - 1/4}$. In order for the solution not to blow up at large z, we set $M_m = 0$. For the massless graviton mode, the remaining solution given in (3.8) reduces to

$$\psi_0 = N_0 \sinh^{1/2}(kz) \cos\left[\frac{\sqrt{3}}{2} \operatorname{arcsinh}(\sinh^{-1}(kz))\right]. \quad (3.10)$$

The trapping of gravity requires that the wave function is normalizable [3], in order to obtain a finite leading-order contribution to the gravitational field on the brane world. We numerically find that $\int dz|\psi_0|^2$ is finite, and the normalization factor $N_0 = 1.07077\sqrt{k}$. As shown in figure 3, the peak of $\psi_0(z)$ is located at $kz \approx .7 \equiv kz_0$. We propose that this is the location of a four-dimensional brane world.[3]

The massive modes oscillate in the bulk, as shown in figure 4. For large kz the massive wave functions given by (3.8) are approximately

$$\psi_m \sim N_m \mathrm{e}^{-i\gamma kz}\,. \quad (3.11)$$

This can be normalized via the delta-function orthogonality condition to yield $N_m = 1/\sqrt{\pi\gamma k}$.

[3]In previous Randall-Sundrum type models, one needed an additional delta-function source in order to localize gravity to the brane. This source was the result of patching together five-dimensional supergravity solutions in a discontinuous manner by taking $z \to c + |z|$, where c is a positive constant. If we extrapolate our solution to $z < 0$, there is already Z_2 symmetry about $z = 0$, which implies that there is a brane world at $z = -z_0$.

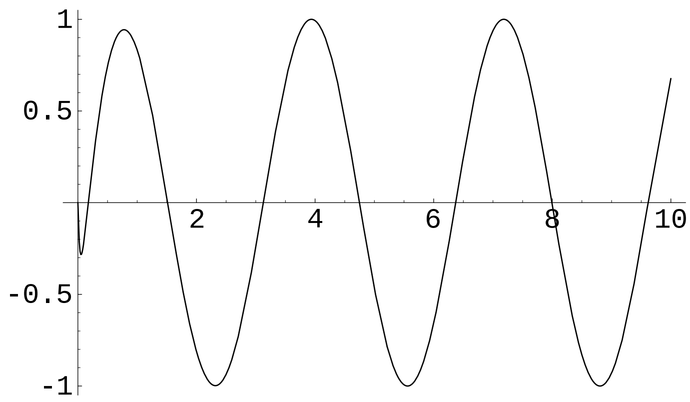

Figure 4: Re $\psi_m(z)$ versus kz for $m = 2k$.

The newtonian gravitational potential between masses M_1 and M_2 can be estimated by [16]

$$U(r) \sim \frac{G_4 M_1 M_2}{r} + \frac{G_5 M_1 M_2}{r} \int_{k^2/4}^{\infty} d(m^2) |\psi_m(z_0)|^2 \mathrm{e}^{-mr} \,. \tag{3.12}$$

The four and five-dimensional Newton constants are related by $G_4 = kG_5$. To find the leading-order correction, we consider $y \equiv m - k/2 \ll k$. In this limit, $|\psi_m(z_0)|^2 \approx .543 N_m^2$. Inserting this into (3.12), we find that

$$U(r) \sim \frac{G_4 M_1 M_2}{r} \left(1 + \frac{.543}{\sqrt{\pi}} \frac{\mathrm{e}^{-\frac{1}{2}kr}}{\sqrt{kr}} \right) . \tag{3.13}$$

The Yukawa-like factor $\mathrm{e}^{-\frac{1}{2}kr}$ in the sub-leading term reflects the presence of a mass gap $\frac{1}{2}k$ separating the zero-energy bound state and the massive Kaluza-Klein continuum.

4. Conclusions

We have shown that the near-horizon of various resolved branes on an Eguchi-Hanson instanton dimensionally reduces to a five-dimensional domain wall that traps gravity. Thus, this Randall-Sundrum type brane world arises from a p-brane origin without an additional delta-function source from imposing Z_2 identification. This brings us substantially closer to embedding infinite extra dimensions in M-theory. Also, this provides us with a brane world model for a strongly-coupled Yang-Mills field theory with quark-antiquark charge screening at finite separation distance.

The Eguchi-Hanson manifold is the simplest example in a class of spaces that are asymptotically locally euclidean (ALE). It may be interesting to see if resolved branes on generic smooth ALE manifolds dimensionally reduce to gravity-trapping domain walls, or whether this is a property of the Eguchi-Hanson instanton in particular.

Also, the question arises as to whether the zero-modes of other bulk fields can be localized to this brane world in a similar manner as for the graviton.

Lastly, as mentioned in the Introduction, a minimally-coupled scalar in the bulk can correspond to either the operator $\mathrm{Tr}\, F^2$ on the gauge theory side (AdS/CFT perspective) or to the graviton fluctuations polarized along the brane worldvolume (Randall-Sundrum perspective). In the IR (small r) regime the spectrum of the glueballs and worldvolume gravitons are expected to be identical [17]. For a confining gauge theory, there is a discrete spectrum of bound states [13]. This would not be desirable from the Randall-Sundrum perspective unless the zero-mode state corresponding to the four-dimensional graviton dominates over the massive bound states, within experimental bounds.

Acknowledgments

We are grateful to Marc Henneaux and Malcolm Fairbairn for useful discussions.

References

[1] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[3] M. Cvetič, H. Lü and C.N. Pope, *Domain walls with localised gravity and domain-wall/QFT correspondence*, *Phys. Rev.* **D 63** (2001) 086004 [hep-th/0007209].

[4] M. Cvetič, M.J. Duff, J.T. Liu, H. Lü, C.N. Pope and K.S. Stelle, *Randall-Sundrum brane tensions*, *Nucl. Phys.* **B 605** (2001) 141 [hep-th/0011167].

[5] A. Karch and L. Randall, *Localized gravity in string theory*, *Phys. Rev. Lett.* **87** (2001) 061601 [hep-th/0105108].

[6] C.S. Chan, P.L. Paul and H. Verlinde, *A note on warped string compactification*, *Nucl. Phys.* **B 581** (2000) 156 [hep-th/0003236].

[7] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[8] I.R. Klebanov and A.A. Tseytlin, *Gravity duals of supersymmetric* $\mathrm{SU}(cn) \times \mathrm{SU}(N+M)$ *gauge theories*, *Nucl. Phys.* **B 578** (2000) 123 [hep-th/0002159].

[9] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and* χ*SB-resolution of naked singularities*, *J. High Energy Phys.* **08** (2000) 052 [hep-th/0007191].

[10] M. Graña and J. Polchinski, *Supersymmetric three-form flux perturbations on* AdS_5, *Phys. Rev.* **D 63** (2001) 026001 [hep-th/0009211].

[11] S.S. Gubser, *Supersymmetry and F-theory realization of the deformed conifold with three-form flux*, hep-th/0010010.

[12] M. Cvetič, H. Lü and C.N. Pope, *Brane resolution through transgression*, *Nucl. Phys.* **B 600** (2001) 103 [hep-th/0011023].

[13] M. Cvetič, G.W. Gibbons, H. Lü and C.N. Pope, *Ricci-flat metrics, harmonic forms and brane resolutions*, hep-th/0012011.

[14] M. Bertolini, V.L. Campos, G. Ferretti, P. Fré, P. Salomonson and M. Trigiante, *Supersymmetric 3-branes on smooth ALE manifolds with flux*, *Nucl. Phys.* **B 617** (2001) 3 [hep-th/0106186].

[15] P. Fré, *BPS D3-branes on smooth ALE manifolds*, hep-th/0110281.

[16] A. Brandhuber and K. Sfetsos, *Non-standard compactifications with mass gaps and Newton's law*, *J. High Energy Phys.* **10** (1999) 013 [hep-th/9908116].

[17] A. Brandhuber and K. Sfetsos, *An $N = 2$ gauge theory and its supergravity dual*, *Phys. Lett.* **B 488** (2000) 373 [hep-th/0004148].